

Exercise 16

Find dy/dx by implicit differentiation.

$$xy = \sqrt{x^2 + y^2}$$

Solution

Differentiate both sides with respect to x .

$$\frac{d}{dx}(xy) = \frac{d}{dx}\sqrt{x^2 + y^2}$$

$$\left[\frac{d}{dx}(x)\right]y + x\left[\frac{d}{dx}(y)\right] = \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \frac{d}{dx}(x^2 + y^2)$$

$$(1)y + x(y') = \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \left[2x + 2y \cdot \frac{d}{dx}(y)\right]$$

$$y + xy' = (x^2 + y^2)^{-1/2}(x + yy')$$

$$y + xy' = \frac{x + yy'}{\sqrt{x^2 + y^2}}$$

$$y\sqrt{x^2 + y^2} + x\sqrt{x^2 + y^2}y' = x + yy'$$

Solve for y' .

$$(x\sqrt{x^2 + y^2} - y)y' = x - y\sqrt{x^2 + y^2}$$

$$y' = \frac{x - y\sqrt{x^2 + y^2}}{x\sqrt{x^2 + y^2} - y}$$